



PREP4UNI | SEMESTER 1 MID-TERM TEST

Calculus: Derivatives

Based on Thomas' Calculus, Chapter 3: Sections 3.1 (Tangent Lines), 3.2 (The Derivative as a Function), 3.3 (Differentiation Rules)

Student name:	_____	Date:	_____
Roll no. / Batch:	_____	Time allowed:	90 minutes
Teacher:	_____	Maximum marks:	60

INSTRUCTIONS TO STUDENTS

- Attempt **all** questions. The paper has three sections (A, B, C) of 20 marks each. Marks for each part are shown in square brackets.
- Show **all working**. Method marks are awarded even when the final answer is incorrect.
- Where a question says *using the limit definition*, the derivative must be found from first principles; the differentiation rules will not earn marks for that part.
- Unless a decimal answer is asked for, leave answers in exact form. A non-programmable scientific calculator is allowed; graphing calculators are not.
- Write your answers on the answer sheets provided. The blank grid for Question 9(b) is printed on this paper; complete it there and hand it in with your answer sheets.

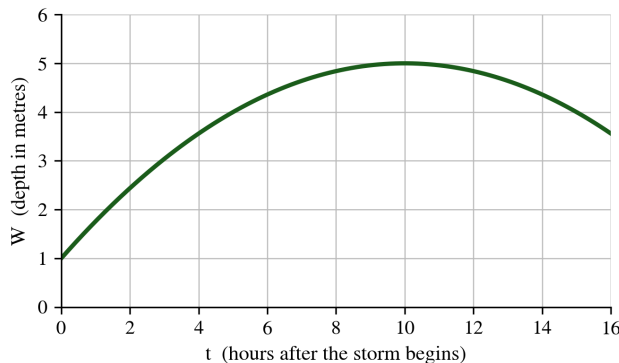
SECTION A: Tangent Lines and the Derivative at a Point (Section 3.1)

[20 marks]

- The graph of $f(x) = \frac{4}{x+1}$ passes through the point $P(1, 2)$.
 - Using the limit definition $f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$, find the slope of the graph of f at P . **[3]**
 - Hence find an equation of the tangent line to the graph at P . **[2]**
- Find an equation of the straight line with slope $\frac{1}{3}$ that is tangent to the curve $y = \sqrt{x}$. **[3]**
- Object dropped from a cliff.** A stone is dropped from the top of an 80 m cliff. Its height above the ground t seconds after release is $h(t) = 80 - 4.9t^2$ metres.
 - Find the average velocity of the stone over the interval $1 \leq t \leq 4$. **[2]**
 - Using the limit definition, find the instantaneous velocity at $t = 2$. State the units and explain what the sign of your answer tells you. **[3]**



4. **Water in a reservoir.** The graph shows the depth W (in metres) of water in a reservoir t hours after a storm begins.



- (a) On which interval does the depth appear to be increasing? What is true about $W'(t)$ on that interval? [2]
- (b) At approximately what time does the depth reach its maximum? What is the value of W' at that time? [1]
- (c) Explain what $W'(6)$ represents, and state its units. [1]
5. **Vertical tangent lines.** Let $f(x) = x^{1/5}$ and $g(x) = x^{4/5}$. By evaluating the limit of the difference quotient at $x = 0$, decide whether the graph of each function has a vertical tangent line at the origin. Give reasons.
- (a) The function f . [1]
- (b) The function g . [2]

SECTION B: The Derivative as a Function (Section 3.2)**[20 marks]**

6. Let $f(x) = \frac{2}{x+3}$.
- (a) Use the formula $f'(x) = \lim_{z \rightarrow x} \frac{f(z) - f(x)}{z - x}$ to find $f'(x)$. [4]
- (b) Find an equation of the tangent line to the graph of f at $x = -1$. [2]

7. **One-sided derivatives.** Consider the function

$$f(x) = \begin{cases} x^2, & x < 1 \\ 3x - 2, & x \geq 1 \end{cases}$$

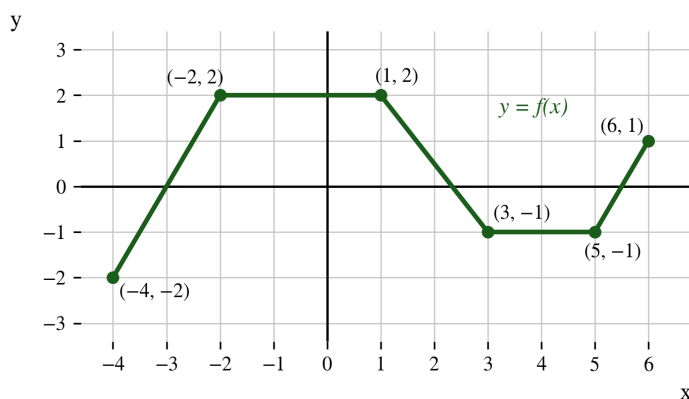
- (a) Show that f is continuous at $x = 1$. [1]
- (b) Compute the right-hand derivative and the left-hand derivative of f at $x = 1$ using limits. [2]
- (c) Is f differentiable at $x = 1$? Give a reason. [1]



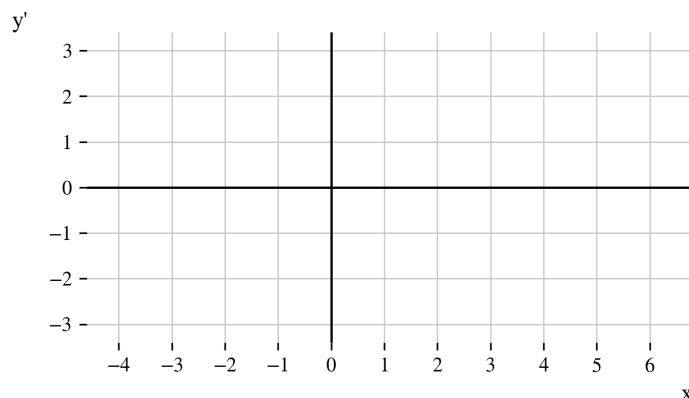
8. **Differentiability on the whole line.** Find the values of the constants a and b that make the function below differentiable for all real x . [4]

$$f(x) = \begin{cases} ax^2 + bx, & x < 2 \\ 3x - 2, & x \geq 2 \end{cases}$$

9. The graph of $y = f(x)$ shown below is made of line segments joined end to end.



- (a) At which points of the open interval $(-4, 6)$ is f' not defined? Give a reason for your answer. [3]
- (b) On the grid below, sketch the graph of $y = f'(x)$. Your graph should be a step function. [3]



Grid for Question 9(b): write your name on this page and hand it in.

**SECTION C: Differentiation Rules (Section 3.3)****[20 marks]**

- 10.** Find the first and second derivatives of $y = \frac{x^4}{2} - 4x^2 + \frac{3}{x} + 9$.
- (a) The first derivative $\frac{dy}{dx}$. [2]
- (b) The second derivative $\frac{d^2y}{dx^2}$. [2]
- 11.** Find the derivative of each function and simplify your answer.
- (a) $y = \frac{3x-2}{x^2+1}$ (use the Quotient Rule). [2]
- (b) $f(t) = (t^2 + 3)\left(t - \frac{1}{t}\right)$ (use the Product Rule). [2]
- 12.** Suppose u and v are differentiable functions of x with $u(1) = 3$, $u'(1) = -2$, $v(1) = -1$, $v'(1) = 4$. Find the values of the following derivatives at $x = 1$.
- (a) $\frac{d}{dx}(uv)$ [1]
- (b) $\frac{d}{dx}\left(\frac{u}{v}\right)$ [1]
- (c) $\frac{d}{dx}(3v - 2u)$ [1]
- 13. Tangent and normal lines.** Consider the curve $y = x^3 - 3x + 2$.
- (a) Find an equation of the tangent line and an equation of the normal line to the curve at the point $(2, 4)$. [3]
- (b) Find all points on the curve where the tangent line is horizontal. [2]
- 14. Tangent lines through an outside point.** Find all points on the graph of $y = x^2$ whose tangent lines pass through the point $(1, -3)$. Write an equation for each such tangent line. [4]

END OF PAPER

For examiner's use	Section A	Section B	Section C	Total
Marks obtained				
Maximum marks	20	20	20	60

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